Pontryagin minimum fuel orbit transfer and docking with model predictive control

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A rendezvous, proximity operations and docking (RPOD) mission is proposed to service and refuel a satellite in geostationary orbit. The two phases of the mission include a minimum fuel orbit transfer from medium Earth orbit to a rendezvous with the target satellite. Pontryagin's minimum principle is used to indirectly solve the two point boundary value problem of the orbit transfer, and proximity operations are continued using model predictive control with appropriate state and control constraints based on common docking safety requirements. The performance of the model predictive controller are investigated based on the different levels of constraint applied to the optimization problem.

I. Introduction and Problem Statement

One of the most prominent areas of interest in the space industry is the infrastructure that supports the Earth orbiting satellites. This infrastructure is becoming extremely important due to the reliance of many services on satellite technology like the Global Navigation Satellite System (GNSS) and weather radar satellites that track changes in Earth's atmosphere.These satellites are already operational and in various orbits around Earth, but many of them were put there decades ago. This means the technology at the time of their orbit insertion was much different than the current state of the art, and therefore lack many of the useful features and capabilities that exist today. Among those shortcomings are the inability to perform corrective maneuvers, a deficit of fuel to perform those maneuvers, and damage due to their relatively long lifespans and the fact that many of these satellites orbit in geostationary orbit, which can lead to radiation damage over extended periods of time. All of these issues offer unique challenges, and it is becoming increasingly obvious that there is a need for some sort of infrastructure and servicing system to aid in keeping these Earth orbiting satellites operational for many years to come.

One recent example of introducing infrastructure to existing satellites comes from SpaceLogistics, a subsidiary of Northrop Grumman. They completed the first commercial on-orbit servicing of a satellite with their Mission Extension Vehicle (MEV)[1]. The job of MEV was to rendezvous with a satellite called Intelsat-901 in geostationary orbit. Once MEV was in proximity to the satellite, it was meant to dock with it in proximity operation by latching onto it's liquid apogee engine, mounted on the satellite such that it points radially outward from Earth. Then, once docked with Intelsat-901, would continue to stay docked and acts as a pseudo-engine for the satellite, performing corrective maneuvers for the satellite until it's lifespan came to an end and disposing of it into a higher orbit located just above geostationary orbit called the graveyard.

In this project, an analogous mission scenario is attempted to develop a realistic implement ion of the control scheme required for a satellite servicing mission such as the one done by MEV. One key aspect that is different is the propulsion system chosen. For the real MEV-1 mission from SpaceLogistics, the engine technology used was an ion propulsion system[1], which produces a continuous low-thrust profile. This manifests in the trajectory of an orbit transfer by a slower, longer path that may span many revolutions around Earth before finally completing the rendezvous to geostationary orbit. For the real mission, this took around four months to complete. Due to the difficulty in obtaining an optimal solution for a multiple-orbit transfer, it is more straightforward (and yet, still plausible) to introduce a high thrust control scheme that uses chemical propulsion instead. This results in a lower fuel efficiency, but allows for higher thrusting capabilities and drastically shorter time of flight.

Another key aspect of rendezvous, proximity operations and docking (RPOD) procedures is the introduction of safety constraints into the problem. Because of the proximity, it is necessary that the mission designer takes great caution when implementing an autonomous control strategy in the presence of delicate and expensive equipment. Pictured below in Figure 1 is a depiction of the MAV docking scenario mentioned previously. Much like docking missions with the international space station (ISS), keep out zones are implemented to ensure whatever control strategy is implemented cannot drive the servicer spacecraft into the satellite it is trying to reach, other than the docking surface.

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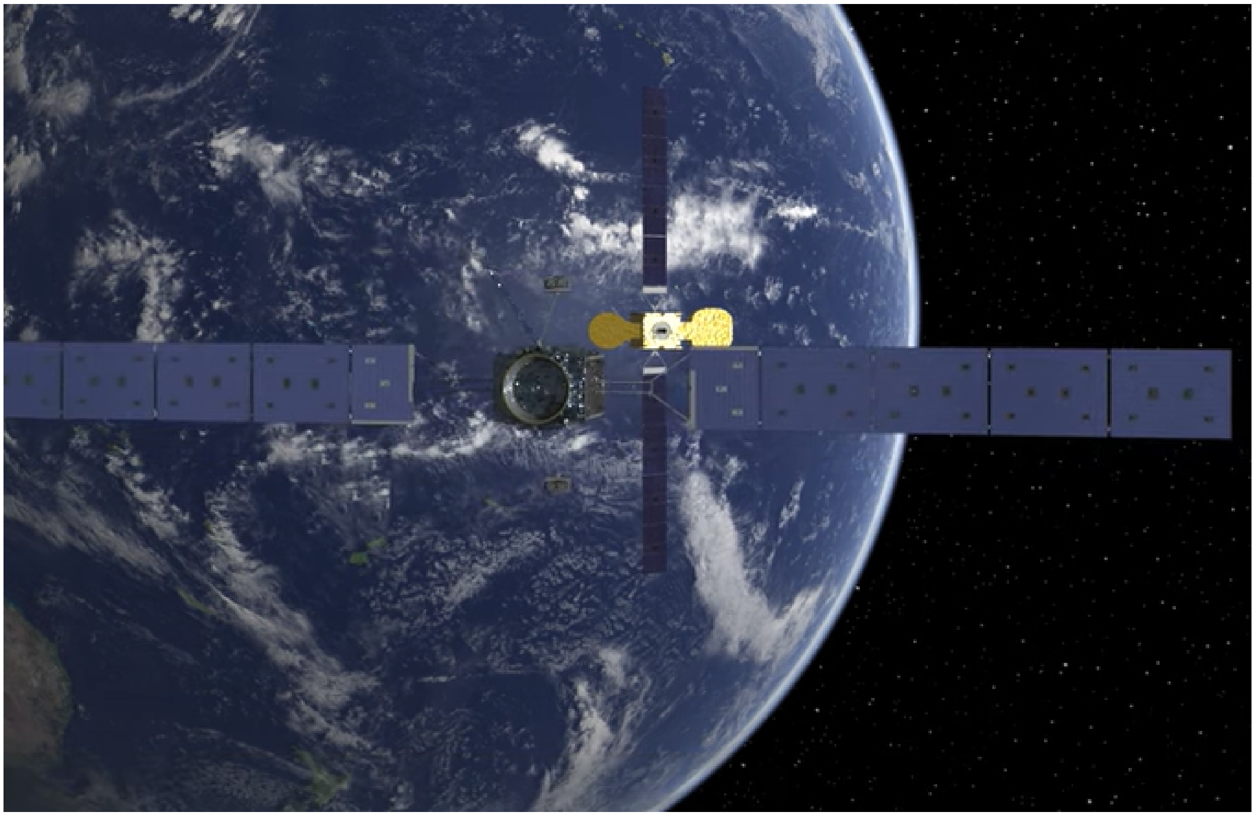

Fig. 1 The MAV-1 Servicer spacecraft approaching the apogee engine of the Intelsat-901 geostationary satellite from the radially outward direction.

This typically involves formulating some kind of geometry that the spacecraft trajectory cannot cross at any time during the mission. In the case of docking with Intelsat-901, the vertical solar arrays take up a large percentage of the feasible space that the servicer spacecraft could potentially cross, so a keep out geometry must be formed to ensure the servicer doesn't collide with these extensions. Also, specific to MAV, are optical sensors like cameras, infrared sensors and light detection and ranging (LiDAR) sensors that work together to ensure precise docking with the target satellite. This is only implemented when in close proximity to the satellite, but it is extremely important that the optical sensors have the target satellite in view upon approach. This is a requirement for any kind of precise docking sequence to be performed, otherwise there is no way that MAV could see the small apogee engine on the back of the satellite that it needs to latch onto. Both of these geometric constraints are absolutely required for mission success, and must be handled in the control of the servicer spacecraft. A mission is planned to transfer the servicer spacecraft from it's parking orbit up into a geostationary orbit, from where is can perform RPOD operations and dock with the Intelsat-901 satellite to take over it's control and thrusting abilities for orbit maintenance and station keeping for the remainder of it's lifespan.

II. Objectives

The tasks of this project are categorized into three distinct levels, with each level indicating an increase in difficulty with respect to both class content and the problem itself. The different sub-tasks at each level are discussed here, and developed further for implementation in the next section.

A. Level 1

As a level one goal, it is necessary to consider a rendezvous scenario between the servicing vehicle and the target satellite. To do this, the respective orbits must be defined, and an orbit transfer is planned to move from one parking orbit to the satellite in geostationary orbit. Due to the nature of the problem, Pontryagin's minimum principle (PMP) is used to determine a minimum-fuel trajectory from the servicer's parking orbit to the geostationary orbit. The indirect shooting method will be used to solve the two point boundary value problem introduced by the PMP problem formulation. The trajectory will be constrained to transversality conditions that stem from the requirements at the initial and final times, as well as a control constraint that places an upper limit on the maximum thrust at any given time in the trajectory.

A minimum-fuel trajectory is sought after due to the propulsion technology of the servicer. Because a high thrust, lower specific impulse chemical engine is used, it is expected that a significant fraction of the onboard fuel reserve will be used in this problem, but the orbit transfer time will be relatively short with a higher thrusting capability. Therefore, the most logical objective function for this orbit transfer is to minimize the fuel consumption, which also implies maximizing the final mass.

B. Level 2

A necessary and natural next step in this mission is to dock with the target satellite in order to obtain control over it's future trajectory and correcting maneuvers, and act as the vehicle's new source of thrust and fuel. In other words, the servicer must come in exact contact with the target satellite at it's docking point without compromising the extensive safety constraints that are normally imposed during proximity operations and docking scenarios. Some of the common safety and operational constraints will include a keep out zone, a line of sight constraint, and the thrust constraint imposed in level 1. A keep out zone is absolutely critical in proximity operations for the safety of both vehicles involved. It ensures that even if something goes wrong in the docking process, the servicer cannot collide with the geostationary satellite. Due to the extensive use of optical sensors such as cameras, infrared sensors and a light detection and ranging (LiDAR) sensor, a line of sight constraint is imposed to ensure the two spacecraft have a view of the docking point in order to ensure a smooth approach and docking execution.

It is well-known that PMP struggles with the addition of state path constraints on top of the control constraints. Because of this, a direct optimization method is used to achieve a trajectory for docking. Even with growing sophistication in hardware onboard spacecraft, the limiting factor of computer resources on a spacecraft is ever present. This motivates the direct optimization method to be formulated in such a way that the optimization can be efficiently executed on-board a servicer spacecraft. While the exact specifications of the hardware limitations are beyond the scope of this project, it will still be a driving factor for the formulation of the direct optimization. Since ours is a nonlinear problem by nature, a nonlinear programming approach is necessary to solve this optimization problem. Keeping the limited resources in mind, it is desirable to derive a quadratic or affine optimization problem that can be directly solved. This will of course introduce some kind of approximation as is common in optimization problems.

While there are many options for this kind of approximating approach, only one control scheme will account for deviation in the expected trajectory. This is why the level 2 goal for this project is to implement a model predictive controller to go from a relatively close position to docking with the target satellite at it's docking point. The MPC allows state and control constraints to be obeyed while still minimizing fuel consumption of the maneuvers required for docking. A computationally efficient formulation of MPC is sought after for this phase of the mission.

C. Level 3

It is known that open-loop control strategies have no guarantee of precision in the objective of the controller. For this reason, the effectiveness of the PMP orbit transfer and the proximity operation control scheme can be evaluated for robustness by introducing a stochastic process to the dynamics of each phase of the mission. By introducing these random variables into the dynamics, a realistic uncertainty of the final state under the control law derived from PMP and MPC can be quantified. This is especially important during the orbit transfer, as the relatively long duration of this phase of the mission can introduce significant state deviation between the two spacecraft, depending on how "uncertain" the dynamics become. This is a common analysis in practice, as it allows the mission designer to see what may happen and what could potentially go wrong.

For these reasons, the level 3 goal of this project is to implement Brownian motion to both the orbit transfer dynamics and the relative motion of the servicer with respect to the target satellite by means of a Monte Carlo simulation. This will involve many consecutive simulations to be ran and tested for robustness after. More specifically, the Brownian motion during the orbit transfer is likely to result in significant state deviation from the target satellite and translate to varying initial conditions for the MPC to handle and try to complete the docking objective anyways. This will give insight into not only the robustness of MPC, but of the level of uncertainty there is in the orbit transfer phase. Even though the Brownian motion would not impact the PMP problem since the stochastic process will be introduced after the PMP problem is solved, it is still useful to see by how much the state may deviate from the expected trajectory during the orbit transfer. The robustness of the MPC may be quantified by mission successes and failures; that is, how often the

MPC will successfully dock with the spacecraft out of the total number of simulations ran.

III. Methods

The first phase of the mission is the orbit transfer problem. As mentioned previously, the goal of this project is to mimic a real application of satellite maintenance and refueling such as the MEV-1 from Northrop Grumman. while the details of the entire mission are not publicly available, it is safe to make some assumptions about this transfer scenario based on similar missions. Unfortunately, there are no universal parking orbits for this type of mission. The most common approach is to enter a LEO injection orbit directly after departure from Earth's surface before performing another maneuver shortly after that begins the transfer orbit into the target geostationary (GEO) orbit[2]. This is done in mission scenarios that involve placing a new satellite from the ground into GEO directly, which is not the mission considered in this project. Unlike those types of orbit transfers, a servicer spacecraft is considered to depart from a higher parking orbit that is likely somewhere in medium Earth orbit (MEO). Much like the GEO orbit transfer problem considered in *Faber et. al.*[3], a parking orbit in the region in between LEO and GEO is considered here. This choice of parking orbit is not fixed, but is typically treated as an optimization parameter in literature that discusses long-term satellite upkeep infrastructure. For the orbit transfer phase, let the problem frame be denoted by $\mathcal{F}$, the Earth equatorial frame. Then the cartesian state vector of the servicer spacecraft and the target satellite are denoted by $\mathbf{x}_s \in \mathbb{R}^6$ and $\mathbf{x}_t \in \mathbb{R}^6$. The initial Keplerian elements of both the servicer's parking orbit and the target GEO orbit are given in Table 1 below. They are converted into $\mathcal{F}$ by a frame transformation, and is then used as the initial state of both spacecraft.

Table 1 The Keplerian orbital elements of the servicer parking orbit and the target geostationary orbit.

Orbit Element	Symbol	Parking Orbit	GEO Orbit	Unit
semi-major axis	a	14,000	42,164.1	km
eccentricity	e	0	0	-
inclination	i	0	0	deg
right ascension of the ascending node	Ω	0	0	deg
argument of periapsis	ω	0	0	deg
mean anomaly	M	310	110	deg

Because this is a minimum-fuel problem, it is necessary to track the used fuel mass of the servicer over the course of the transfer by introducing the state variable m into the state vector $\mathbf{x}_s = \begin{bmatrix}\mathbf{r}^\top & \mathbf{v}^\top & m\end{bmatrix}^\top \in \mathbb{R}^7$. Pontryagin's minimum principle[4] can be used to define the conditions of optimality when considering a problem that is formulated as a two point boundary value problem (TPBVP). The state dynamics are given in Equation 1.

$$\mathbf{f}(\mathbf{x}, \mathbf{u}) = \dot{\mathbf{x}} = \begin{bmatrix}\dot{\mathbf{r}} \\ \dot{\mathbf{v}} \\ \dot{m}\end{bmatrix} = \begin{bmatrix}\mathbf{v} \\ -\frac{\mu}{r^3}\mathbf{r} + \frac{1}{m}\mathbf{u} \\ -\alpha\|\mathbf{u}\|_2\end{bmatrix} \tag{1}$$

where μ is the gravitational parameter of Earth, and α is related to the specific impulse and the fuel efficiency of the propulsion system onboard the servicer spacecaft. Next, the control Hamiltonian is formed based on the minimum-fuel problem type. The Lagrangian to be minimized is

$$\mathcal{L} = \|\mathbf{u}\|_2$$

by definition. Therefore, the control Hamiltonian is formulated as

$$H = \mathcal{L} + \boldsymbol{\lambda}^\top \mathbf{f}(\mathbf{x}, \mathbf{u}) = \|\mathbf{u}\|_2 + \boldsymbol{\lambda}^\top \mathbf{f}(\mathbf{x}, \mathbf{u})$$

Now, with a control Hamiltonian defined it is natural to derive the costate dynamics next according to PMP.

$$\dot{\boldsymbol{\lambda}}^\top = -\boldsymbol{\lambda}^\top \nabla_{\mathbf{x}} \mathbf{f}(\mathbf{x}, \mathbf{u}) \tag{2}$$

Taking the gradient of the state dynamics with respect to the state yields

$$\nabla_{\mathbf{x}}\mathbf{f}(\mathbf{x},\mathbf{u}) = \begin{bmatrix} 0_{3\times3} & I_{3\times3} & 0_{3\times1} \\ -\frac{\mu}{r^3}I_{3\times3} + \frac{3\mu}{r^5}\mathbf{r}\mathbf{r}^\top & 0_{3\times3} & -\frac{\mathbf{u}}{m^2} \\ 0_{1\times3} & 0_{1\times3} & 0 \end{bmatrix} \in \mathbb{R}^{7\times7} \tag{3}$$

Next, the optimal control law derived from PMP is found. It is required to find an optimal control $\mathbf{u}^*$ that minimizes the control Hamiltonian within the set of admissible control, that is, $\mathcal{U} \in \{\mathbf{u} : \mathbf{u} \leq u_{\max}\}$, where $u_{\max}$ is derived from the capabilities of the servicer spacecraft's control authority.

$$\mathbf{u}^* = \arg\min_{\mathbf{u}\in\mathcal{U}} H(\mathbf{x}^*, \mathbf{u}, \lambda)$$

Let Γ be the optimal control magnitude, and $\hat{\mathbf{u}}$ be the optimal control direction unit vector. Then

$$\Gamma, \hat{\mathbf{u}} = \arg\min\left[\|\mathbf{u}\|_2 + \boldsymbol{\lambda}_r^\top\mathbf{v} + \boldsymbol{\lambda}_v^\top\left(-\frac{\mu}{r^3}\mathbf{r} + \frac{1}{m}\mathbf{u}\right) - \lambda_m\alpha\|\mathbf{u}\|_2\right]$$

where $\boldsymbol{\lambda}_r, \boldsymbol{\lambda}_v, \lambda_m$ are the position, velocity and mass costates, respectively. The minimization argument can be further reduced to individually solve for the optimal magnitude and direction.

$$\Gamma, \hat{\mathbf{u}} = \arg\min\left[\Gamma\left(1 + \boldsymbol{\lambda}_v^\top\frac{\hat{\mathbf{u}}}{m} - \lambda_m\alpha\right)\right]$$

The minium value of the argument depends on the sign of the term inside the parentheses. Regardless of the overall sign, first consider the optimal direction, that is, the direction that maximizes the magnitude of the term inside the parentheses. The term $\boldsymbol{\lambda}_v^\top\frac{\hat{\mathbf{u}}}{m}$ is maximized when the control direction is the same as the velocity costate since the term is equivalent to an inner product between the velocity costate and $\frac{\hat{\mathbf{u}}}{m}$. The optimal control direction is given by

$$\hat{\mathbf{u}} = -\frac{\boldsymbol{\lambda}_v}{\|\boldsymbol{\lambda}_v\|_2} \tag{4}$$

which will guarantee that when the control input is non-zero, the term inside the parentheses is as negative as possible. Following the same logic, it is desirable to minimize the argument by maximizing the magnitude of the argument by maximizing Γ whenever the term inside the parentheses is negative, and minimize the magnitude of Γ whenever it is positive. This logic is expressed in Equation 5 below.

$$\Gamma = \begin{cases} u_{\max}, & \text{if } S < 0 \\ 0, & \text{if } S > 0 \end{cases} \qquad S = 1 - \frac{1}{m}\|\boldsymbol{\lambda}_v\|_2 - \lambda_m\alpha \tag{5}$$

Because this equation for Γ is discontinuous, the numerical solver that will be implemented (and discussed later) to solve the TPBVP will struggle with this definition of the optimal control magnitude. Because of this, it is necessary to approximate the optimal control magnitude with a smoothing function given in Equation 6.

$$\Gamma \approx \frac{u_{\max}}{2}\left[1 + \tanh\left(\frac{S}{\rho}\right)\right] \tag{6}$$

where ρ is the sharpness parameter. Lastly, the transversality conditions for the orbit transfer minimum-fuel problem must be determined. Because this is a rendezvous mission, the final positions and velocities of the servicer spacecraft and the satellite must be the same.

$$\psi = \begin{bmatrix} \mathbf{r}_s - \mathbf{r}_t \\ \mathbf{v}_s - \mathbf{v}_t \end{bmatrix}$$

It is assumed that both the initial and final times are fixed. The reason for the final time being fixed is arbitrary, since a real application would likely depend on the mission requirements, e.g., is it necessary that rendezvous occurs at a certain anomaly or epoch, or should the servicer get there as soon as possible? For the sake of simplicity, this implementation will have a fixed final time. This means the only additional transversality condition to consider is the one in which the final state is not fixed, i.e., $\mathrm{d}t_f \neq 0$.

$$\boldsymbol{\nu}^\top \psi_{t_f} - \boldsymbol{\lambda}(t_f)^\top = 0 \implies \boldsymbol{\nu}^\top \begin{bmatrix} I_{3\times3} & 0_{3\times3} & 0_{3\times1} \\ 0_{3\times3} & I_{3\times3} & 0_{3\times1} \end{bmatrix} - \boldsymbol{\lambda}^\top(t_f) = 0 \implies \lambda_m(t_f) = 0$$

Then the overall required transversality conditions are given by Equation 7.

$$\Psi(\mathbf{z}) = \begin{bmatrix} \mathbf{r}_s - \mathbf{r}_t \\ \mathbf{v}_s - \mathbf{v}_t \\ \lambda_m(t_f) \end{bmatrix} \in \mathbb{R}^7, \quad \mathbf{z} = \lambda_0 \tag{7}$$

The PMP orbit transfer is then indirectly solved by a single shooting method in MATLAB while obeying the conditions of optimality for this problem given by Equations 1-7.

Next, the model predictive controller is formulated. First, a new frame C, the relative motion frame, is introduced due to the proximity of the servicer with respect to the target satellite. At this range, the linearized dynamics of this motion is accurate. Since GEO orbits are circular, the dynamics can be further simplified without any loss of accuracy to the well known Clohessy-Wiltshire-Hill (CWH) frame given by Equation 8, and is shown in Figure 2 as well.

$$\dot{\mathbf{x}}_s = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 3n^2 & 0 & 0 & 0 & 2n & 0 \\ 0 & 0 & 0 & -2n & 0 & 0 \\ 0 & 0 & -n^2 & 0 & 0 & 0 \end{bmatrix} \mathbf{x}_s + \begin{bmatrix} 0_{3\times3} \\ I_{3\times3} \end{bmatrix} \mathbf{u} \tag{8}$$

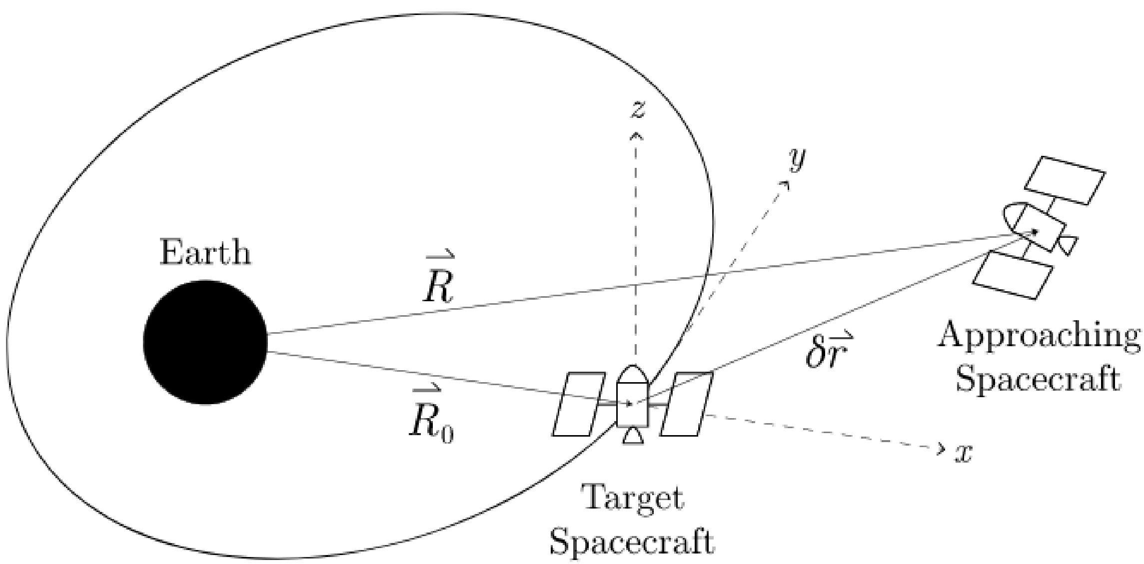

Fig. 2 The CWH frame used for the dynamics during the proximity operations near the target satellite.

where n is the mean motion of the GEO satellite. The minimum fuel objective is still present in the proximity operations by trying to minimize the control authority over the optimal trajectory. In a similar formulation to existing MPC schemes[5][6], it is desirable to obtain a quadratic programming problem if possible due to the limited computer resources onboard a small spacecraft like the servicer spacecraft. In the spirit of attempting to formulated a quadratic optimization problem, a finite-horizon linear quadratic regulator (LQR) cost function is derived. Before deriving the rest of the MPC control problem, it is necessary to first linearize and discretize the equations of motion. In the case of CWH equations of motion, the dynamics are already affine, so only the discretization needs to be done. The integrals used to discretize the motion of the servicer spacecraft between time steps are

$$A_k = \Phi(t_{k+1}, t_k)$$

and

$$B_k = \int_{t_k}^{t_{k+1}} \Phi(t_{k+1}, \tau) \mathrm{d}\tau$$

where $\Phi(\cdot)$ is the state transition matrix. Then the discretized dynamics are given by Equation 9.

$$\mathbf{x}_{k+1} = A_k \mathbf{x}_k + B_k \mathbf{u} \tag{9}$$

Because the dynamics are linear time invariant, the discretization of the dynamics only needs to happen once before the control problem starts, and each time the horizon or time step of the MPC changes. Now, the optimization constraints are considered next. In a similar fashion to the PMP orbit transfer problem, there is an upper bound on the admissible control given by

$$||\mathbf{u}||_\infty \leq u_{\max} \tag{10}$$

The two critical path constraints come in the form of geometrical boundaries, similar to other proximity and docking literature[5]. The line of sight (LoS) constraint ensures the docking can enforce precise correction when in very close proximity by having optical sensor data during the approach. To enforce this field of view constant for the optical sensors, a line of sight cone is enforced, and is described by Equation 11.

$$\sqrt{y^2 + z^2} \leq x \tan\theta \tag{11}$$

where θ is the half-angle of the LoS cone. as discussed in previous sections, the MEV-1 mission to Intelsat-901 requires a radially outward approach direction (approaching the target satellite in a direction radially inward toward Earth), defined by the positive x axis in the C frame, which is the reason for the form of the LoS constraint, which has the cone starting at the target satellite and pointing outward toward the approach and docking face of the satellite. This formulation is automatically infeasible when the initial condition in the MPC problem is located outside of the LoS cone. To address this feasibility issue, the LoS constraint is imposed in the cost function with a relatively high weight, which has virtually the same effect as a true path constraint, but with feasibility for a more realistic range of initial conditions. The other issue comes from the fact that $x \in \mathbb{R}$, meaning it can take any real number value. This makes the plan LoS unbounded in an optimization sense, since x can be chosen to be ∞ to enforce the constraint, of course resulting in an invalid solution to the optimization problem. To address this, a slack variable is introduced to lift the LoS constraint. Let $s \in \mathbb{R}^0$ (nonnegative real numbers) be the slack variable, which is also included in the cost function with a large weight value. Then the LoS "soft" constraint becomes

$$\sqrt{y^2 + z^2} \leq x \tan\theta + s \tag{12}$$

This tells the optimizer that s should be minimized with great effort, and as a consequence of this, x must meet the original constraint in order to stay feasible. This prevents the unboundedness seen before with the introduction of only one slack variable. Lastly, the safety-critical keep out zone constraint is imposed next. Due to the general shape of the target satellite that is to be avoided, an ellipsoid is a simple and accurate representation of the keep out zone that needs to be strictly enforced. Due to the fact that safety is critical in RPOD scenarios, this path constraint must be a hard constraint that must absolutely be enforced at all times no matter what. This means it cannot be added as a term in the cost function, but must be directly addressed as a constraint in the optimization problem, choosing infeasibility over constraint violation. The ellipsoid keep out zone (KOZ) is described below in Equations 13-14.

$$n^\top \mathbf{r}_s \geq n^\top \mathbf{r}_0 \tag{13}$$

$$n = 2S\mathbf{r}_0 \tag{14a}$$

$$\mathbf{r}_0 = \eta d \tag{14b}$$

$$\eta = (\mathbf{d}^\top S \mathbf{d})^{-1/2} \tag{14c}$$

$$S = \mathrm{diag}(r_x, r_y, r_z) \tag{14d}$$

Here, r_x, r_y, r_z are the axis lengths of the ellipsoid that defines the KOZ, and $\mathbf{d}$ is the reference position vector, which allows the linear approximation of the KOZ at each point in the servicer spacecraft's trajectory. This means the KOZ constraint is linearized depending on the relative position of the spacecraft. This is due to the hyperplane that the linearization represents, which creates a hyperplane linear constraint each new discrete position in the optimal trajectory. The reference vector $\mathbf{d}$ is determined based on the last iteration of the MPC horizon solution, and initialized as a straight line from the servicer spacecraft's relative position and the origin (i.e., the target satellite). This provides a linear hard constraint that ensures the servicer may never entire the ellipsoid without becoming an infeasible optimization problem, which is a requirement for important safety constraints like this one. With the path constraints discussed, there is one more constraint to consider. The initial condition must be enforced at each horizon evaluation. This is simply done by an affine equality constraint, enforcing the initial condition to equal the actual initial condition of the MPC problem at the first horizon, and at each new horizon, the initial condition is updated to be the last "measured" real state of the servicer spacecraft. This is only a simulation of the controller, so instead, the full nonlinear relative equations of motion are numerically integrated for the length of the timestep chosen, and the final state from the integration is used as the next horizon's initial condition.

The final condition would normally be enforced in a direct optimization problem as well, however, the MPC does not guarantee that the horizon reaches the completion of the docking. Due to the fact that MPC is a receding horizon control algorithm, enforcing a hard final time constraint to be docked with the target satellite will almost certainly result in infeasibility, especially at the beginning of the problem when the servicer is farther away from the target satellite. To avoid any kind of drifting and motivate the optimizer to converge on the target satellite quickly, another term is added as a kind of "soft" constraint to the cost function. This comes in the same form as the quadratic term in the receding horizon LQR that penalizes state deviation (in this case, the state since the reference is the origin, or the target satellite in the C frame), but with significantly larger weight to make sure the optimizer chooses trajectories that drive the servicer to the target rapidly, even if the horizon doesn't see the final goal yet. With all constraints considered, the cost function is the last piece to be included, covering any soft constraints that were previously introduced. The cost function is given in Equation 15.

$$J = \sum_{k=1}^{N} [\ \mathbf{x}_k Q \mathbf{x}_k + \mathbf{u}_k R \mathbf{u}_k + w s_k\ ] + \mathbf{x}_N T \mathbf{x}_N \tag{15}$$

Now with Equations 8-15 fully defining the constrained proximity and docking phase of the mission, the implementation of the entire mission is ready to be done in MATLAB.

IV. Results

The problem parameters introduced in the orbit transfer phase are summarized in Table 2 below, followed by the plot of the results from the minimum-fuel orbit transfer to GEO orbit shown in Figures 3-4. Note that $\alpha = \frac{1}{g_0 I_{sp}}$, where g_0 is the gravitational acceleration at sea level.

Table 2 The problem parameters for the MEO to GEO orbit transfer phase.

Parameter	Symbol	Value	Unit
specific impulse	I_{sp}	320	s
maximum thrust	u_{max}	0.225	kN
sharpness parameter	ρ	1×10^{-4}	-
final time (nd)	t_f	2.5	-

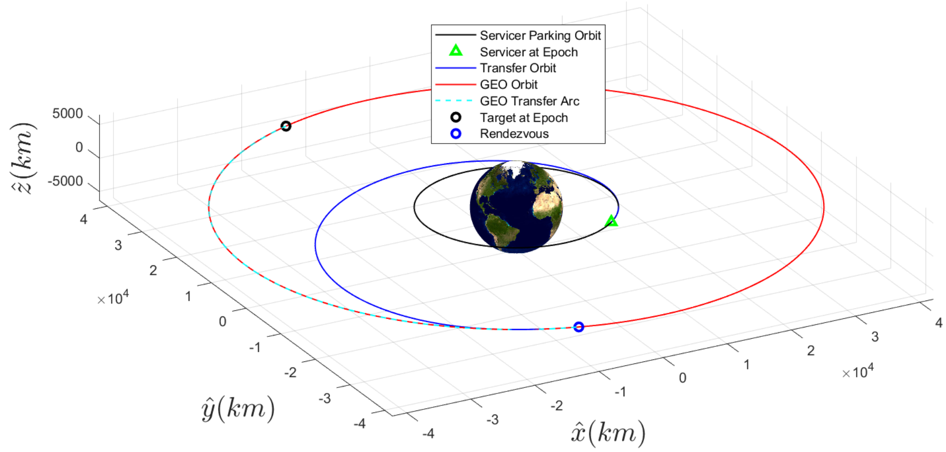

Fig. 3 The isometric view of the orbit transfer from MEO to GEO in the Earth equatorial frame.

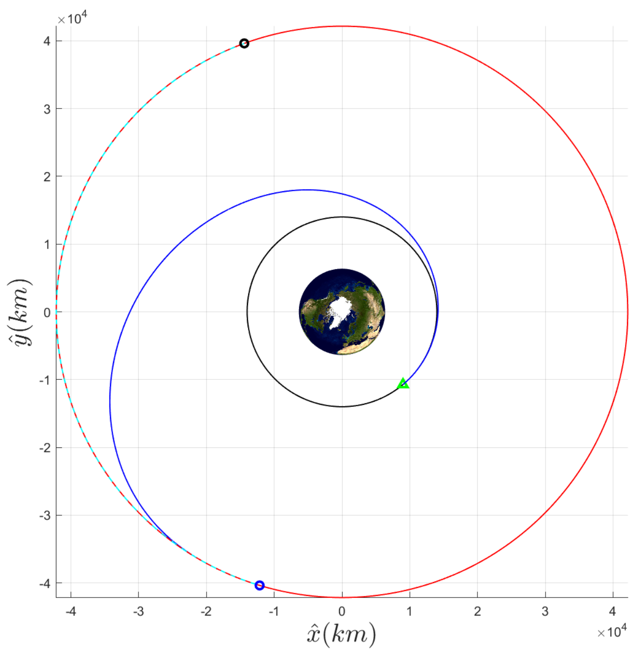

Fig. 4 The top-down view of the orbit transfer from MEO to GEO in the Earth equatorial frame.

The resulting control profile for the orbit transfer, and the corresponding deviation in the control Hamiltonian are shown next in Figures 5-6. Recalling the PMP conditions of optimality, the control Hamiltonian is minimized for the optimal control profile. This means a metric for the "optimality" of the result is the deviation of the control Hamiltonian over the course of the trajectory.

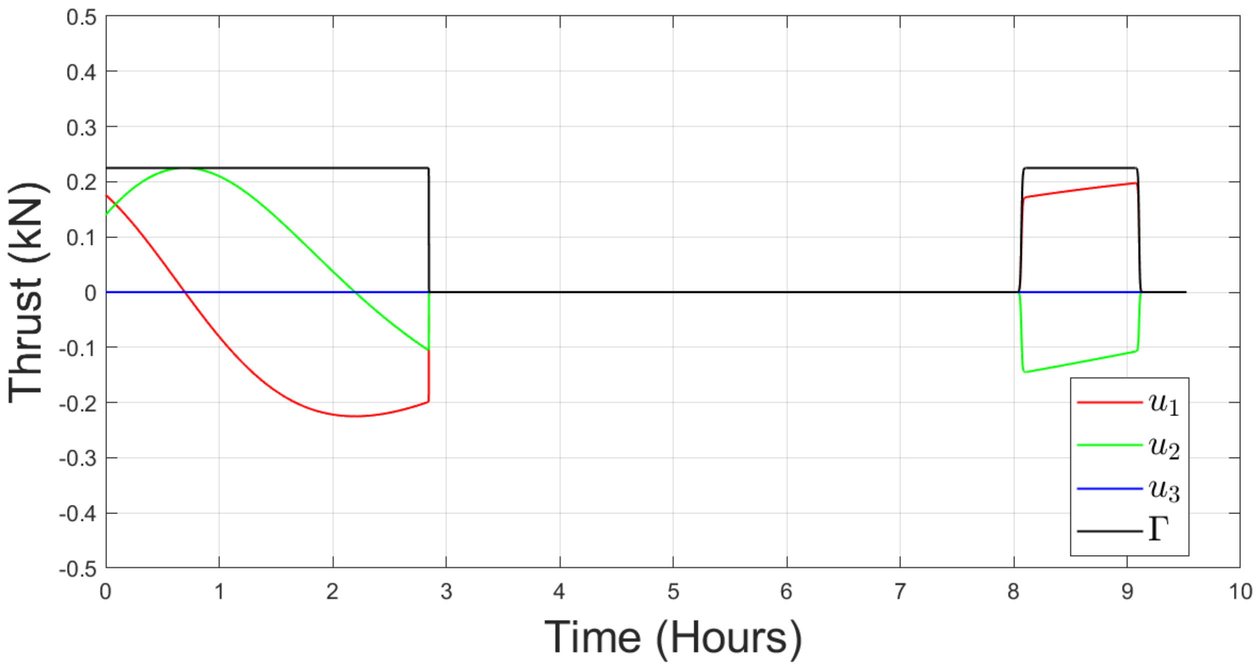

Fig. 5 The optimal minimum-fuel control profile for the orbit transfer phase, including the individual thrust vector elements and the absolute magnitude.

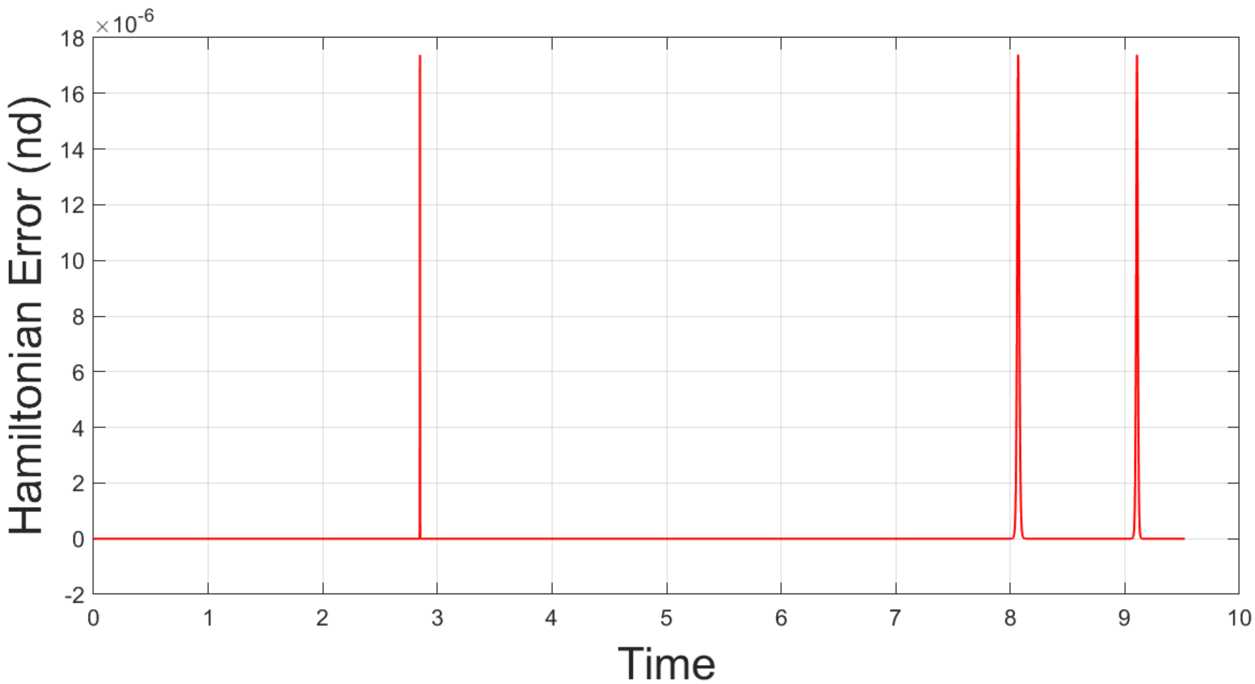

Fig. 6 The deviation of the control Hamiltonian over the duration of the orbit transfer.

For the PMP orbit transfer to GEO orbit, the optimal control law is approximated with a smoothing function. Many approaches to initial costate guesses exist, including the common approach of starting with a minimum-energy problem, and using it's initial costate solution as the initial guess in the minimum-fuel problem. In this case, it was decided to use a successive costate guess approach using different sharpness parameter values of increasingly small values. As the sharpness parameter ρ becomes smaller and smaller, the shape of the approximated smoothing function converges to the discontinuous optimal control profile. The successive approach allows easier convergence for the indirect shooting method in finding an initial costate guess for the minimum-fuel problem. The final initial costate result from the PMP orbit transfer problem is

$$\boldsymbol{\lambda}_0 = \begin{bmatrix} -2.7821 \\ 3.3897 \\ 0.0000 \\ -0.6802 \\ -0.5409 \\ -0.0000 \\ 0.6112 \end{bmatrix} \tag{16}$$

Before implementing the MPC phase of the mission, one more modification was made to ensure the controller's performance. The time step and horizon length chosen initially does not work well for close proximity operation. This is mainly because the maneuvers are required to be more precise, and the relative state changes much faster with respect to the time step and horizon of the original formulation. In other words, there is too much going on and too many things that need to be controlled in elapsed time for each time step. For this reason, a small portion of logic is modified in the MPC. After the next control input is applied and the dynamics are propagated forward for one time step, the range of the servicer spacecraft from the target satellite is checked. If the range is within 50 meters from the target, then a proximity operation flag is set, and the timestep is reduced further, with a longer horizon for the next iteration. Then, the discrete-time matrices A_k and B_k found earlier are updated to reflect the changes in the time step. The optimization problem is initialized in the next iteration with these new MPC parameters to reflect the faster control law when in close proximity to the target.

Through the entire trajectory of the MPC phase, a hard path constraint in the form of an ellipsoidal keep out zone is imposed in the optimization problem at each time step. This is a hard constraint and cannot be violated at any point. However, this hard constraint will make the problem infeasible with a complete guarantee when the servicer spacecraft attempts the final approach into docking to the target satellite, since it has to enter the KOZ in order to completely dock with the target. To address this issue, the KOZ is enforced until the range of the servicer is within 6 meters of the target satellite. This number is chosen so that the hard constraint is enforced up until the very final approach phase, where the servicer must enter the ellipsoid from the radially positive direction, which has a radius on the ellipsoid of 5 meters (based on the rough geometry information of the Intelsat-901). When the range is less than 6 meters, the KOZ is no longer enforced in the optimization, allowing the servicer to enter the ellipsoid KOZ upon the final approach. The combination of these two logical changes in the MPC have provided a smooth transition between the range phases of the MPC problem, and maximize the effectiveness of both state path constraints.

To show the effect of the path constraints on the proximity operations, different iterations of the MPC problem are ran with and without the state path constraints. The initial condition was purposely chosen to showcase the possibility of danger when the MPC has no path constraints to begin with. The initial state of the servicer spacecraft in the proximity phase is

$$\mathbf{x}_0 = \begin{bmatrix} -0.75 \\ 0 \\ 0.005 \\ 0.003 \\ 0.009 \\ -0.004 \end{bmatrix}$$

in the C frame. The initial position is such that the default trajectory approaches the target satellite from the radially outward direction, meaning in order to dock with the GEO satellite, it may try to cross directly through it causing a collision. For the MPC and it's respective phases, the parameters defining the horizon and the controller speed are described below in Table 3.

Table 3 The initial MPC parameters.

Parameter	Symbol	$\|\mathbf{r}\|_2 > 50$m	$\|\mathbf{r}\|_2 < 50$m	Unit
time step	Δt	3	2	s
horizon length	N	30	50	-

The general proximity phase parameters for the MPC are also given below in Table 4, including the weights for each penalty term in the cost function.

Table 4 The problem parameters for the MPC problem including the weights for each penalty term in the cost function.

Parameter	Symbol	Value	Unit
maximum thrust	u_{max}	0.225	kN
position penalty	Q_{pos}	100	-
velocity penalty	Q_{vel}	50,000	-
control penalty	R_{ctrl}	10	-
terminal penalty	T_{N}	100,000	-
line of sight weight	w	100	-
ellipsoid x axis radius	r_x	5	m
ellipsoid y axis radius	r_y	8	m
ellipsoid z axis radius	r_z	20	m
theta	θ	15	deg

In this form, $Q = \text{blkdiag}(Q_{\text{pos}} I_3, Q_{\text{vel}} I_3)$, $R = R_{\text{ctrl}} I_3$, and $S = \text{diag}(r_x^{-2},\ r_y^{-2},\ r_z^{-2})$. For the first case, only the initial condition and control constraints are imposed, meaning there are no KOZ constraints or LoS constraints. The results are shown below in Figures 7-9.

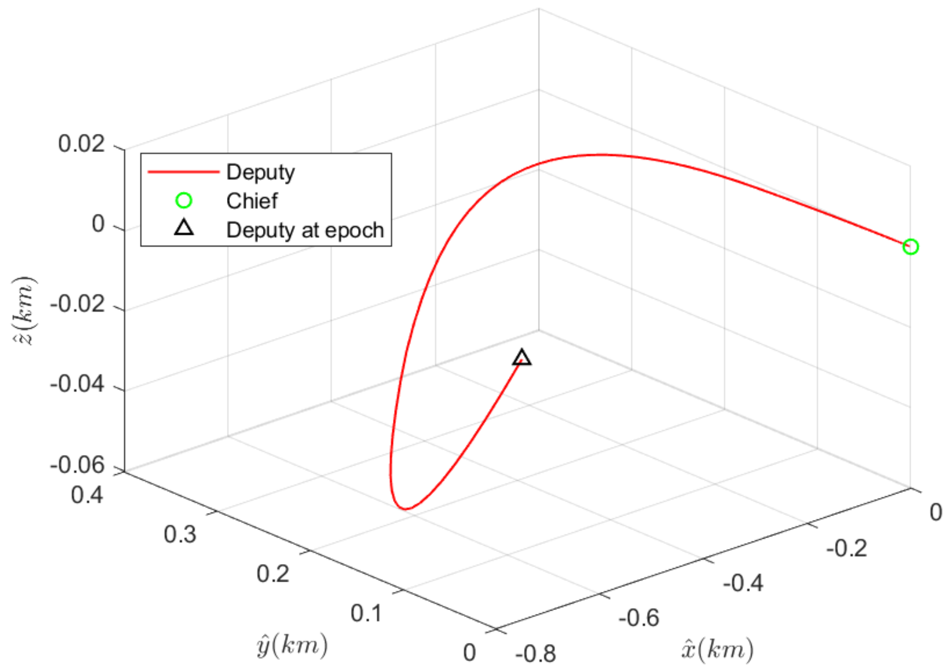

Fig. 7 The servicer spacecraft trajectory in the C frame with no state path constraints. In this frame, the servicer spacecraft is denoted as "deputy", while the target GEO satellite is the "chief", following conventional nomenclature in the CWH frame.

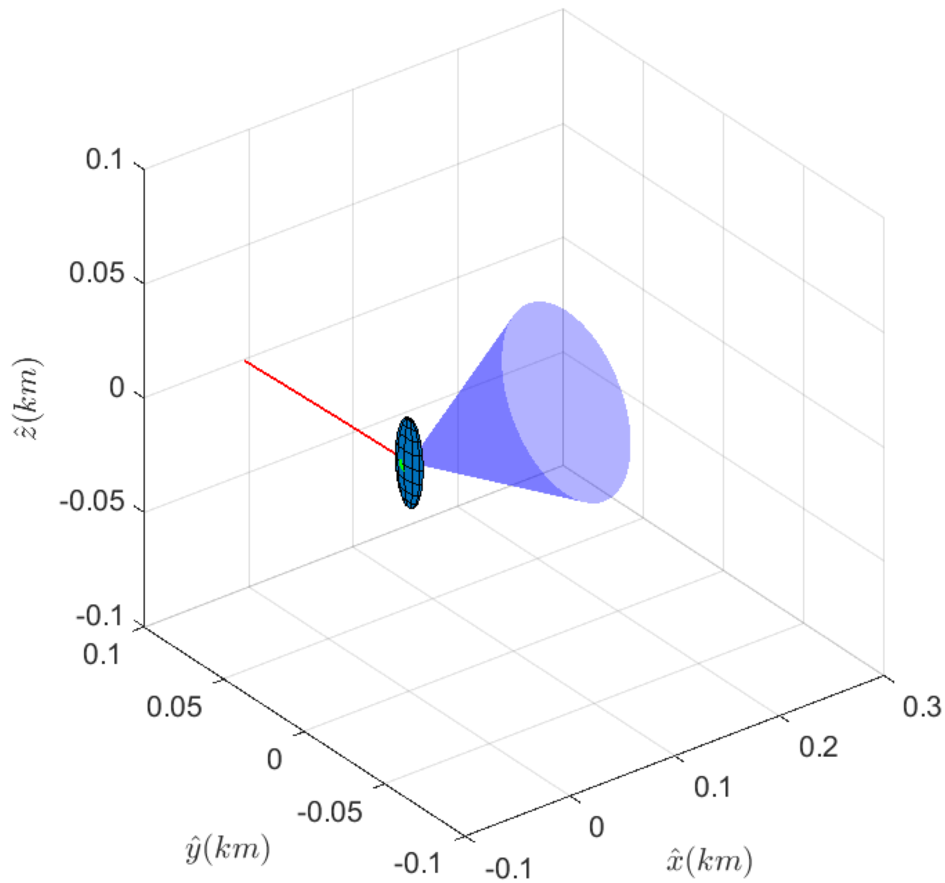

Fig. 8 The relative trajectory zoomed in at the docking range, depicting the KOZ and LoS cone as well.

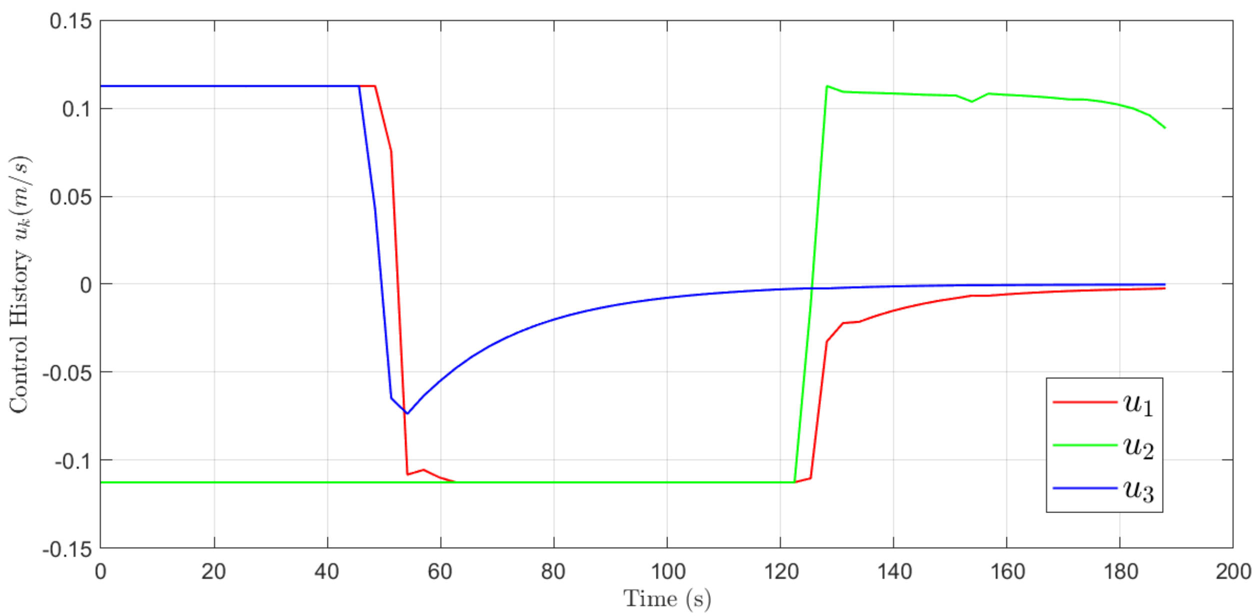

Fig. 9 The control history for the proximity phase with no constraints.

An identical control scheme is performed again, this time including the LoS soft constraint and the KOZ constraint as well. The results are shown below in Figures 10-12.

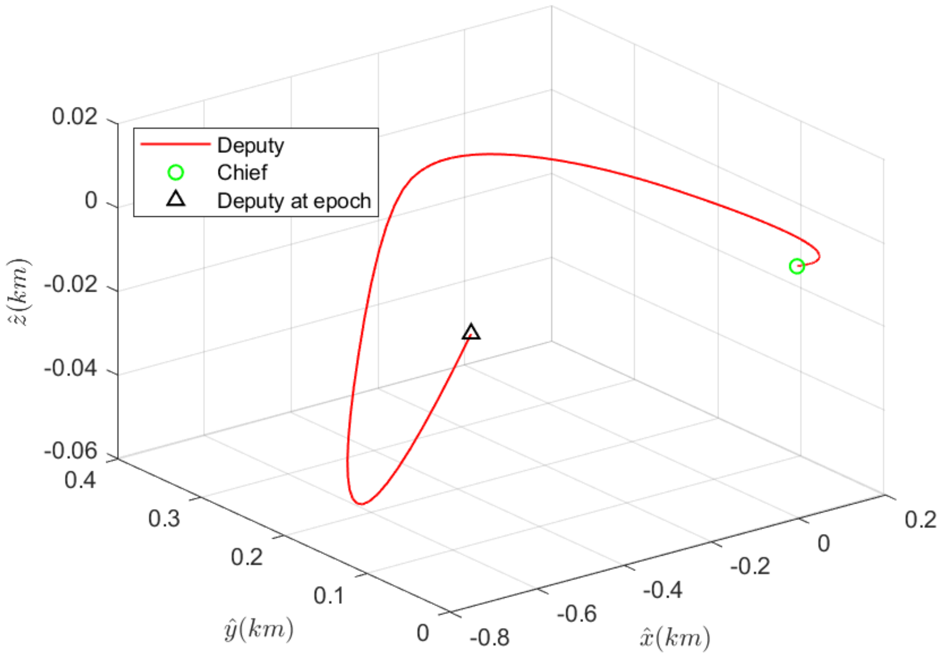

Fig. 10 The servicer spacecraft trajectory in the C frame with KOZ and LoS state path constraints. In this frame, the servicer spacecraft is denoted as "deputy", while the target GEO satellite is the "chief", following conventional nomenclature in the CWH frame.

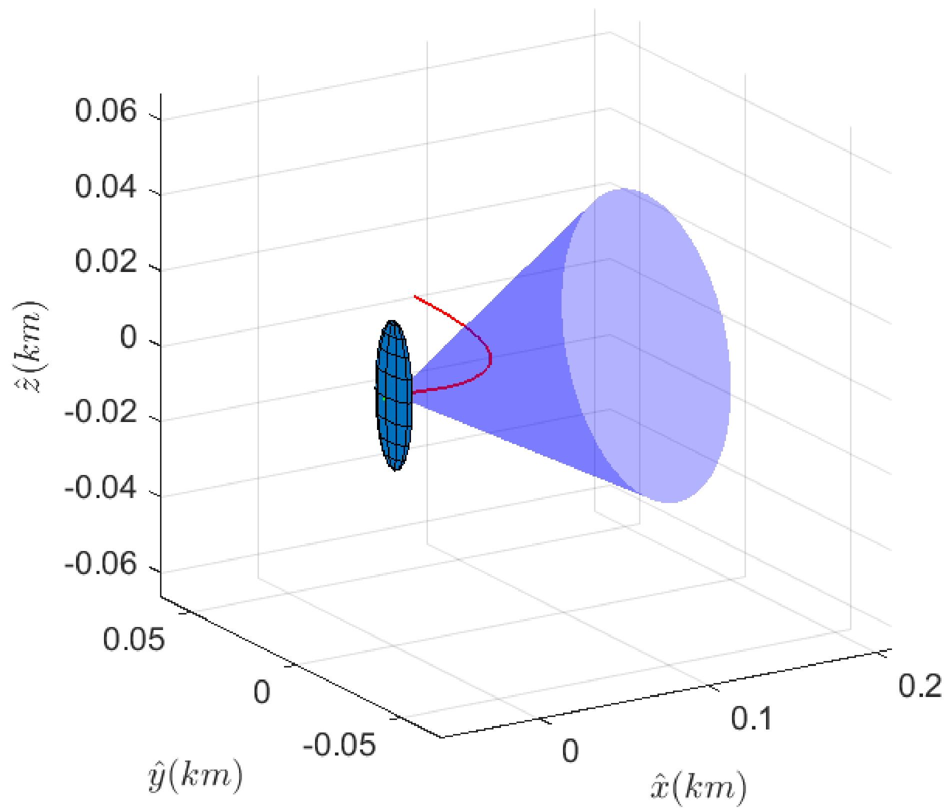

Fig. 11 The relative trajectory zoomed in at the docking range, depicting the KOZ and LoS cone as well.

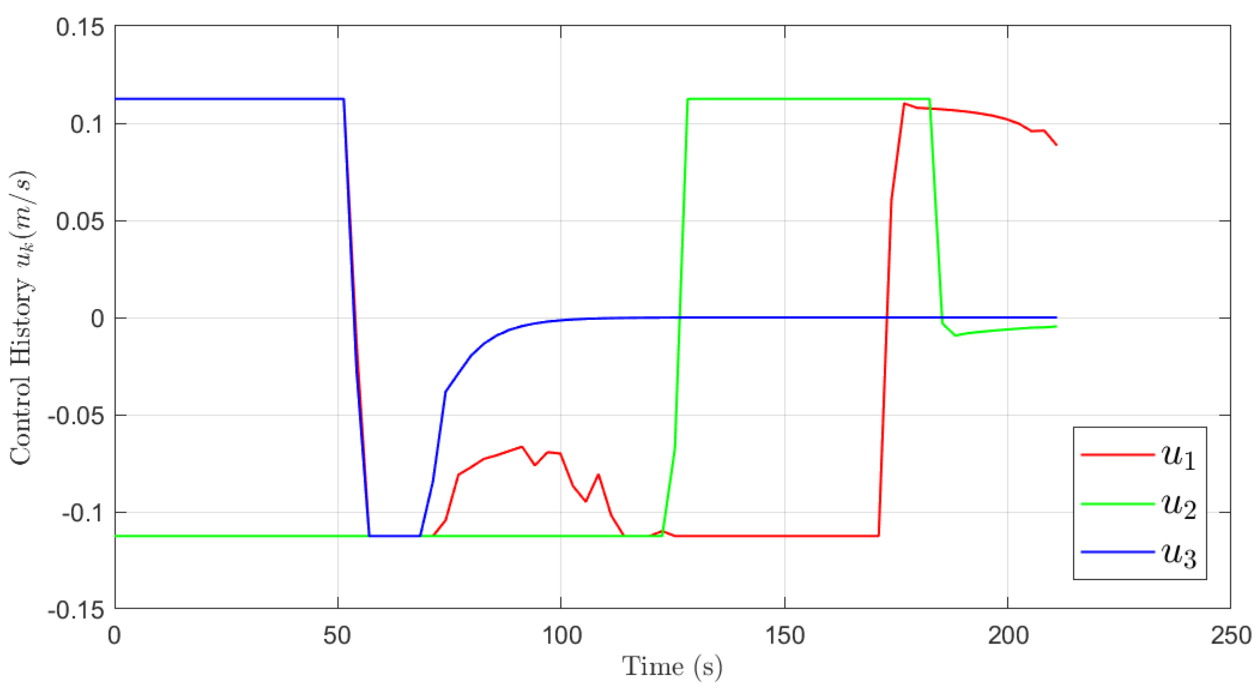

Fig. 12 The control history for the proximity phase with KOZ and LoS constraints.

The total delta-v, elapsed mission time and fuel mass spent for each phase of the mission and the mission as a whole are presented below in Table 5.

Table 5 The final results of the RPOD mission to rendezvous and service a GEO satellite. The delta-v delivered, elapsed time and spent fuel mass are shown for each phase individually, and in total for the entire mission.

Result	Orbit Transfer Phase	Proximity Phase	Total	Unit
delta-v	2.1834	0.0349	2.2183	km/s
elapsed time	571.392	3.733	575.125	minutes
fuel mass spent	1,002.36	11.03	1,013.39	kg

V. Discussion

Generally speaking, the results from both phases of the mission point to success. While there are a few points of concern, the general requirements of the project were met, and the numerical values obtains as results are within reason. For the orbit transfer phase, one metric for success was of course the rendezvous. From the spatial plots, it is clear to see the two trajectories intersect in GEO orbit, presumably circularizing during the last few minutes of the transfer orbit. This is also reflect in the control profile shown in Figure 5. As expected, the controller magnitude switches between fully on and off, or a bang-bang control profile. This is a known result of a minimum-fuel problem, and typically coincides with the orbit transfer that was done in the project. For reference, the mass consumption was approximately 50% of the total mass. The mass of the vehicle was not considered for simplicity, but assuming the vehicle started with $2,000$ kg of fuel in reverse for the mission, 50% seems high for an orbit transfer that takes roughly 10 hours to complete. Because of the minimum-fuel objective, the control profile resembles impulsive maneuvers, which makes sense for a high-thrust propulsion system. Further exploring that idea, the results were checked with a Hohmann transfer between the same two orbits. This is due to the similarity between this transfer and the Hohmann transfer. There are two circular orbits, and the transfer angle of this one 200 deg. Since the Hohmann transfer provides the smallest delta-v for this kind of orbit transfer for two impulsive maneuvers, it serves as an excellent baseline of comparison for the PMP orbit transfer. The results found in the project were a total delta-v of 2.1834 km/s across three maneuvers, while the Hohmann transfer result results in a delta-v of 2.1061 km/s. Since these costs are so similar, it is perfectly reasonable that the delta-v for the PMP orbit transfer is correct and by extension the fuel mass consumption as well.

For the second phase of the mission and the level 2 goal of the project, the results were also satisfactory. Of course, it is expected that the delta-v and elapsed time for this phase should be much smaller than that of the orbit transfer phase since the scale is so much smaller. Even with a significant deviation in the initial state from the target satellite, the MPC provides a constrained control scheme that obeys strict safety constraints and also attempts to minimize the used fuel, evident in the results of this phase of the mission. The consumed fuel mass was around 11 kg, and suggests the only maneuvers needed were small in magnitude and likely only due to course correction for the overall path. This is also evidence to suggest the MPC parameters were appropriately chosen as well. Before presenting the final results for the project, other combinations of Δt and N, the finite horizon length, were attempted. In some cases, similar results were obtained with small fuel mass consumption, while others were clearly not ideal as the optimizer tries to make trajectories without enough, (or with too much) knowledge of the future trajectory. In some cases, this lead to overshooting and other convergence issues, especially in the close proximity phase near the target satellite. From the first MPC implementation that had no path constraints, this trajectory would lead to a collision between the spacecraft and satellite, and even with the KOZ, would likely not reach the appropriate docking side of the target satellite. It is clear from the path-constrained result that both constraints are necessary for a clean and safe docking sequence to occur.

There are two major improvements I would have liked to attempt for this project. The first applies to the orbit transfer phase, with the assumption that was made about the propulsion system. For the real MEV-1 servicer spacecraft, an ion propulsion system was used instead. This is a low-thrust engine, which means the control strategy would be much different than this impulsive profile obtained in the project. The ion propulsion would be much more realistic, but as previously mentioned, is not trivial to solve with Pontryagin's minimum principle. A more sophisticated guessing scheme for the initial costate would likely be needed to be able to solve the transfer with Pontryagin's minimum principle, or perhaps something more simple like a Lyapunov controller instead for the multiple orbit transfer with a low thrust profile.

Secondly, the MPC was sufficiently realistic for this project and for the goal of docking with the Intelsat-901 at it's apogee engine, but further constraints could be introduced to make the problem even more realistic. For example, an important constraint to consider with chemical engines and cold gas maneuvering systems is plume impingement on the surface of the target spacecraft. If some kind of significant mass is expelled during maneuvering, then this may damage sensitive surfaces, optical sensors, or other items on board the target satellite during the close proximity approach. The thrusting direction during the close proximity range must then be further constrained so that the plume from the servicer spacecraft's engines does not hit the target satellite. This is especially important since the project proposed here uses a chemical propulsion system that has a large amount of plume that is generated during maneuvering. Also, a small detail was assuming that the LoS constraint originates from the target satellite, which implies the optical sensors are mounted on the target and not the servicer spacecraft. From MEV-1, this assumption is wrong; Intelsat-901, for example, has no actual docking hardware or sensors on board, but the servicer was designed to mount onto it's apogee engine instead[1]. This becomes another area of improve in the MPC formulation, which is attaching the origin of the LoS constraint to the servicer spacecraft's docking face instead of the target. For simplicity, this detail was purposely omitted from the project but could be reformulated in future iterations to provide a more realistic scenario if the docking is based on the MEV-1 to Intelsat-901 mission.

The level 3 goal for this project was not completed, but certain measures were taken to ensure that levels 1 and 2 could be completed independently of level 3. If the final level had been attempted, it would have included Brownian motion during both phases, or at least the orbit transfer phase, and would be used to make the problem more realistic, and add another metric of success for the proximity phase. If time permitted, level 3 would likely be implemented by solving the PMP problem exactly as it was done in level 1, but propagated using the resulting open-loop control trajectory from PMP and including the stochastic process as well. This simulates the uncertainty that mission designers encounter when designing transfers, as there are unknown perturbations and possible even model uncertainty for the transfer phase. As long as there is a way to reliably quantify this uncertainty, an orbit transfer followed by a proximity phase with MPC can account for any reasonable stochastic process implemented. The results of the stochastic differential equations being propagated forward in time would be used as initial conditions for the same MPC controller developed in level 2. Then, this would act as a Monte Carlo simulation for the MPC, and would be based on realistic results that come from the orbit transfer.

VI. Conclusions

In completing this project, I have learned a lot about rendezvous, proximity operations and docking procedures, specifically about the constraints that are required for this kind of problem. In practice, many implementations do not exceed classical control theory like PID controllers, but with increasingly capable computational resources onboard spacecraft, the feasibility of having online optimal control strategies are growing. The main takeaway from the level 1 goal was the difficulty in proposing a realistic problem with Pontryagin's minimum principle. In theory, the methods in PMP aren't too complex, but coming up with realistic scenarios in which it can be used for GEO orbit transfer are difficult if realistic hardware or servicing spacecraft designs are considered. In hindsight, it may have been worth considering another approach for the orbit transfer phase of the mission. In contrast, I imagine it would be difficult to formulate the proximity and docking control without using some kind of model-based control. This is because of the hard safety and path constraints. While, for example, it is possible to use something like Lyapunov control to enforce constraints, they cannot be guaranteed without significant analytical efforts. This makes MPC an intuitive choice for control when handling delicate and safety-oriented operations between two spacecraft such as the problem posed here. Specifically with the proximity operation, it is important to take into consideration the edge cases in which more constraints may be necessary. For example, I thought it may have been sufficient using only a LoS constraint for the proximity phase, however, when considering the initial conditions that started directly opposite from the target destination, it becomes clear that hard safety constraint can prevent a collision. Even though the odds of this happening are relatively low, a good philosophy when working with RPOD is to over-plan and over-deliver, especially on safety requirements. I would suggest to anyone pursuing a similar project to implement more constraints than necessary to ensure safety. It is better to have a simple objective or problem, but with thorough and well planned constraints and redundancies in place as a general rule of design. Overall, in terms of the scope of this project, all requirements for levels 1 and 2 were met, and with time permitting level 3 could have been implemented with some extra effort. Future work for a similar project may involve introducing further constraints to the MPC, and considering other control schemes for the orbit transfer phase of the mission.

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